Fundamental Study for a Fractional Order Repetitive Control Using Generalized Repetitive Control for High Precision Motor Control

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***Abstract*— Interior Permanent Magnet Synchronous Motors(IPMSMs) are widely used for industrial applications. IPMSMs has a lot of advantages, such as high efficiency, high torque density and so on. In such a drive system, harmonic current appears inevitably and harmonic current control technique is used for the compensation. Repetitive control is known as a useful harmonic current compensation method and realized by adding a specific number of a sampling period according to the frequency of repetitive disturbance. Such a discrete delay must be an integer of sampling period. However, it has fractional term in a specific frequency region of the disturbance.**

This paper proposes a novel fractional order repetititve control using Generalized Repetititve Control(GRC) ,which is based on Generalized KYP lemma proposed by S.Hara. GRC is compared with Lagrange interpolation method, which is used for fractional order repetitive control conventionally. The effectiveness of the proposed designing method is verified through simulations and the experiments.

I. INTRODUCTION

In industrial applications, there exist periodic disturbances which are difficult to make modelling. For example, harmonic current in motor drives and periodic disturbance in HDD drive are famous as a factor to prevent high precision control. Harmonic currents in Permanent Magnet Synchronous Motor(PMSM) are generated by the deadtime of the inverter, the offset of the current sensor, harmonic component of the inductance, and so on. They should be suppressed by control algorithms.

Generally, it can be realized to make the tracking error zero by adding the same dynamics of the reference signal. This theory is known as Internal Model Principle(IMP). Based on this theory, various control techniques are proposed. The References [1]-[4] applied IMP for harmonic current control application firstly.

Harmonic current controls are categorized as integrator based methods in synchronous frame or resonant compensators methods in static frame. Integrator based methods in synchronous frame are the method to use synchronous frame coordinate and integrate feedback signal in that coordinate. This control method can be seen as a resonant compensator in static frames. Multi-vector control [5] and Adaptive Feedforward Cancellation [6] are typical examples. The other one is the controller in static frame and it is called as resonant compensators or repetitive controller[7].

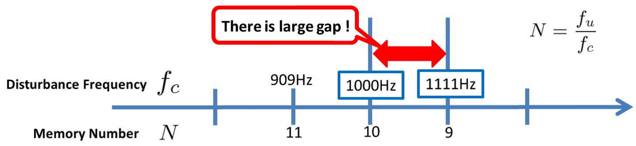


Fig. 1. Fractional order problem in repetitive control system

Repetitive controllers are widely used in industrial applications because of the simplicity. They store the measured data in memories and the memory number N is decided by disturbance frequency f_c and sampling frequency f_u.

$$N = \frac{f_u}{f_c} \tag{1}$$

The memory number needs to be integer, but it happens to be selected as a fractional order according to the disturbance frequency. This is one of the cause which deteriorates position accuracy especially in high frequency region. The relationship between disturbance frequency f_c and memory number N is shown in Fig 1 in the condition that sampling frequency T_u is 10kHz. This problem is known as the name of Fractional Delay(FD) in Signal Processing[14]. FD is conventionally approximated as a FIR filter by Lagrange interpolation method. However, it is difficult to design FD and Low Pass Filter(LPF) ,which is used to robust stability, simultaneously. Furthermore, the approximation accuracy of the Lagrange interpolation is bad in high frequency region. [16] and [15] are examples which apply FD for repetitive control scheme.

In this paper, a fractional order repetitive control(FORC) using generalized repetitive control(GRC)[8][9][10] is proposed. This method uses Generalized KYP lemma and solves LMI problems. The meaning of the design parameter of LMI is very clear for designers. This method enables designers to design FORC systematically and intuitively. The effectiveness is verified by the comperison between Lagrange interpolation method and GRC method.

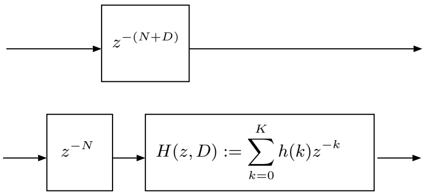


Fig. 2. The concept of Lagrange interpolation

II. LAGRANGE INTERPOLATION METHOD FOR FRACTIONAL ORDER REPETITIVE CONTROL

Fractional Delay(FD) can be approximated by Eq.(2) by Lagrange interpolation method.

$$H(z,D) := \sum_{k=0}^{N_1} h(k,D) z^{-k} \tag{2}$$

$$h(k,D) := \prod_{l=0, l \neq k}^{N_1} \frac{D-l}{k-l} \tag{3}$$

$D(0 < D < 1)$: delay number of fractional delay, N_1 : the order which is used in Lagrange interpolation. The Lagrange interpolation with $N_1 = 1$ corresponds to linear interpolation. The calculation result of the Lagrange interpolation in condition that N_1 =1, 2 and 3 is shown in Table II. Fig. 2 shows the concept of FD by Lagrange interpolation.

III. THE ABSTRACT OF GENERALIZED REPETITIVE CONTROL(GRC)

Generalized Repetitive Control was proposed by G.Pipeleers, J.Swevers, et al, in KULeuven[8]. it is superior to the conventional repetitive control in the following point.

· It can select the memory number freely.
· Desirable characteristics, including the trade-off between the performance of periodic disturbance rejection and of non-periodic disturbance deterioration, can be implemented through Generalized KYP lemma.

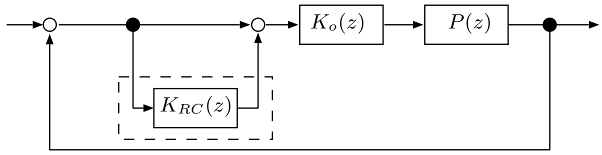
It has a great advantage to shape the sensitivity function in all frequencies.

A. Block diagram of GRC

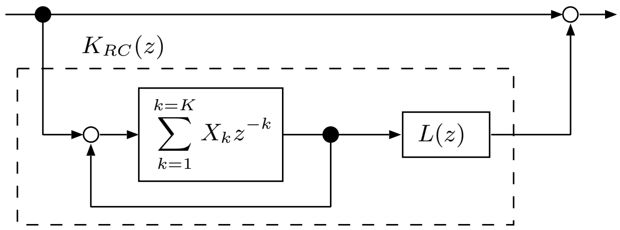
GRC is designed by solving LMI problems. Block diagram of the controller is shown in Fig. 3. Sensitivity Function $S_o(z)$ and complementary sensitivity function $T_o(z)$ without repetitive control is expressed by Eq. (4) and (5).

$$S_o(z) = \frac{1}{1 + P(z)K_o(z)} \tag{4}$$

$$T_o(z) = \frac{P(z)K_o(z)}{1 + P(z)K_o(z)} \tag{5}$$



(a) Block diagram



(b) The detail of $K_{RC}(z)$

Fig. 3. Block diagram of Generalized Repetitive Control

Sensitivity function $S(z)$ with repetitive control is shown by Eq.(6).

$$S(z) = S_o(z) M_S(z) \tag{6}$$

$$M_S(z) := \frac{1}{1 + K_{RC}(z) T_o(z)} \tag{7}$$

In Eq. (6), $M_S(z)$ is the term which shows the change of the sensitivity function with and without repetitive controller. It is called as Modifying Sensitivity Function.

$K_{RC}(z)$ is designed in the following steps. Firstly, $K_{RC}(z)$ is expressed by Eq. (9).

$$X(z) := \sum_{k=1}^{K} X_k z^{-k} \tag{8}$$

$$K_{RC}(z) = \frac{X(z)}{1 - X(z)} L(z) \tag{9}$$

Substituting Eq. (8) and (9) for Eq.(7) leads Eq.(10).

$$M_S(z) = \frac{1 - X(z)}{1 - X(z)[1 - L(z)T_o(z)]} \tag{10}$$

Here, Eq. (11) is true when $L(z)$ is designed as Zero Phase Error Tracking Control(ZPETC)[17] for $T_o(z)$.

$$M_S(z) = 1 - X(z) \tag{11}$$

Eq. (12) and (13) are state space representation for $X(z)$ and $M_S(z)$.

$$\begin{gathered} X(z) \rightarrow^{\text{ss}} \\ A = \begin{bmatrix} 0_{K-1,1} & I_{K-1} \\ 0 & 0_{1,K-1} \end{bmatrix}, B = \begin{bmatrix} 0_{K-1,1} \\ 1 \end{bmatrix} \\ C_1 = \begin{bmatrix} X_K & X_{K-1} & \dots & X_1 \end{bmatrix}, D_1 = \begin{bmatrix} 0 \end{bmatrix} \end{gathered} \tag{12}$$

N_1	h(0)	h(1)	h(2)	h(3)
1	1-D	D	-	-
2	(D-1)(D-2)/2	-D(D-2)	D(D-2)/2	-
3	-(D-1)(D-2)(D-3)/6	D(D-2)(D-3)/2	-D(D-1)(D-3)/2	D(D-1)(D-2)/6

TABLE I

FIR FILTER COEFFICIENT BY LAGRANGE INTERPOLATION (N_1 = 1, 2, 3)

$$M_S(z) \rightarrow^{\text{ss}}$$
$$A = \begin{bmatrix} 0_{K-1,1} & I_{K-1} \\ 0 & 0_{1,K-1} \end{bmatrix}, B = \begin{bmatrix} 0_{K-1,1} \\ 1 \end{bmatrix}$$
$$C_2 = \begin{bmatrix} -X_K & -X_{K-1} & \dots & -X_1 \end{bmatrix}, D_2 = \begin{bmatrix} 1 \end{bmatrix} \quad (13)$$

B. formularization of the characteristics for GRC

Considering the requirement of harmonic current control, GRC needs to have the following three characteristics.
1. stability in high frequency region
2. small deterioration of the sensitivity function at non-periodic disturbance frequency
3. large improvement of the sensitivity fucntion at periodic disturbance frequency
These characteristics is formulated as Eq.(14)-(17).

$$X^* = \text{argmin}\gamma_{p,\Delta} + \alpha\gamma_{np} \quad (14)$$

subject to

$$|X(\omega)| \leq \varepsilon, \quad \forall\omega \geq \omega_\varepsilon \quad (15)$$
$$||M_S(\omega)||_\infty \leq \gamma_{np} \quad (16)$$
$$V_l|M_S(\omega)| \leq \gamma_{p,\Delta}, \quad \forall\omega \in \omega_l \quad (17)$$
$$\Omega_l = [l\omega_0(1-\Delta), l\omega_0(1+\Delta)]$$

Figure 4 shows the definition of these parameters.

C. LMI formulation of the frequency characteristics

Gain characteristics in finite frequency can be converted to necessary and sufficient LMI conditions through GKYP lemma Prof. Hara and Iwasaki proposed in [11]. The following LMIs can be obtained by applying GKYP lemma to Eq.(15) - (17).
Eq. (15)⇔
There exist Hermitian matrices P_1 and Q_1 such that

$$\begin{bmatrix} A^T P_1 A - P_1 & A^T P_1 B & C_1^T \\ B^T P_1 A & B^T P_1 B - \varepsilon^2 I & D_1^T \\ C_1 & D_1 & -I \end{bmatrix} + \begin{bmatrix} -Q_1 A - A^T Q_1 + 2\cos(\omega_\varepsilon T_u) Q_1 & -Q_1 B & 0 \\ -B^T Q_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \leq 0 \quad (18)$$

$$Q_1 \geq 0 \quad (19)$$

Eq. (16)⇔
There exist symmetric matrices P_2 such that

$$\begin{bmatrix} A^T P_2 A - P_2 & A^T P_2 B & C_2^T \\ B^T P_2 A & B^T P_2 B - \gamma_{np}^2 I & D_2^T \\ C_2 & D_2 & -I \end{bmatrix} \leq 0 \quad (20)$$

Eq.(17)⇔
There exist Hermitian matrices P_3 and Q_3 such that

$$\begin{bmatrix} A^T P_3 A - P_3 & A^T P_3 B & C_2^T \\ B^T P_3 A & B^T P_3 B - \gamma_{p,\Delta}^2 I & D_2^T \\ C_2 & D_2 & -I \end{bmatrix} + \begin{bmatrix} -\eta_2 Q_3 A - \eta_1 A^T Q_3 + \eta_3 Q_3 & \eta_2 Q_3 B & 0 \\ \eta_1 B^T Q_3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \leq 0 \quad (21)$$

$$Q_3 \geq 0 \quad (22)$$

Here, $\eta_1 := e^{j\omega_0 T_u}, \eta_2 := e^{-j\omega_0 T_u}, \eta_3 := \cos(\omega_0 \Delta T_u)$ and P_i, Q_j are symmetric matries. GRC can be calculated to minimize the objective function expressed in Eq. (14) in the LMIs of (18)-(22). In this paper, the LMIs is described by YALMIP[18] and this optimization is solved by SDPT3[19]. Note Eq. (21) includes complex values. This constraint is converted to real LMI problems through the equivalence of $X + jY > 0$ and

$$\begin{bmatrix} X & Y \\ -Y & X \end{bmatrix} > 0. \quad (23)$$

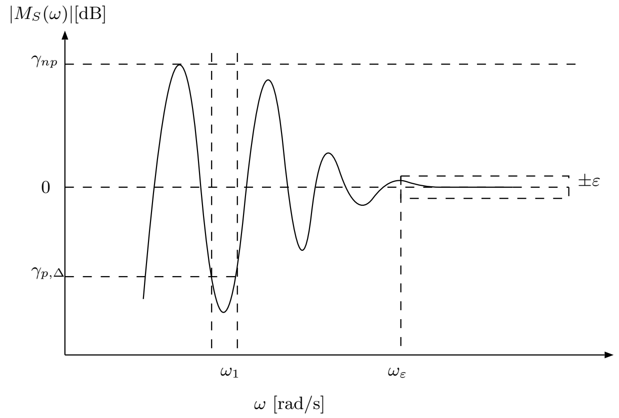
Here, X and Y are real square matrices representing the real and imaginary parts of the complex LMI. See the reference [20] for further details.

IV. COMPARIZON BETWEEN THE PERFORMANCE OF CONVENTIONAL AND PROPOSED METHOD

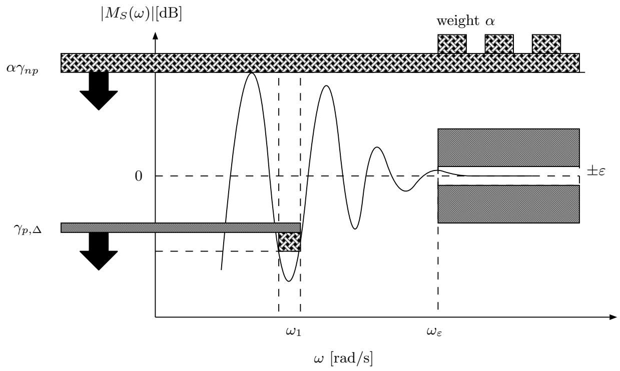
In this section, a fractional order repetitive controller(FORC) which is designed by Lagrange interpolation and Zero Phase LPF (this is called as conventional method) and the other one which is designed by GRC (this is called as proposed method) is compared. These two controllers have the same memory number and calculation algorithm. Only filter coefficients differ.

A. Conventional Method : FROC designed by Lagrange interpolation and Zero phase LPF

When FROC is designed by the conventional method, the parameter N and D is determined by sampling frequency f_u



(a) The definition of LMI design parameter



(b) The concept of the sensitivity optimization procedure throught GKYP lemma

Fig. 4. The concept of LMI design paramter and optimization problem

and disturbance frequency.

$$N + D = \frac{f_u}{f_c} \tag{24}$$

$$N := \text{int}\left[\frac{f_u}{f_c}\right] \tag{25}$$

By Using N and D, conventional controller is expressed as Eq. (26).

$$H(z) = z^{-N} H(z, D) Q(z) \tag{26}$$

$$Q(z) := \left(\frac{z + \gamma + z^{-1}}{\gamma + 2}\right)^{N_2} \tag{27}$$

In Eq. (26), the degree of Lagrange interpolation N_1 and the Zero phase LPF parameter of γ, N_2 are selected by controller designers.

B. Proposed method : FORC of GRC

Eq.(26) is transformed to (28).

$$X(z) = \sum_{k=N-N_2}^{N+N_1+N_2} X'_k z^{-k} \tag{28}$$

In the proposed methods, filter coefficients X'_k are calculated by LMI optimization problem. The conventional method and the proposed method calclate the output by the same algorithms. The difference between these two methods exists in the filter coefficients which are used in that calculation.

C. Simulation Result

In this paper, Sampling frequency f_c is 10kHz, and 6th and 12th components of the harmonic current is selected to be periodic disturbances for motor drive applications. This means ω_0 is the frequency of 6th order harmonic current and l = 1, 2 are considered in Eq. (17). In Design 1, f_c is 488Hz, and it means N is 20 and D is 0.5. In Design 2, f_c is 952Hz, and it means N is 10 and D is 0.5. To make a fair comparizon, the H infinity norms γ_{np} of the modifying sensitivity function are equalized and the

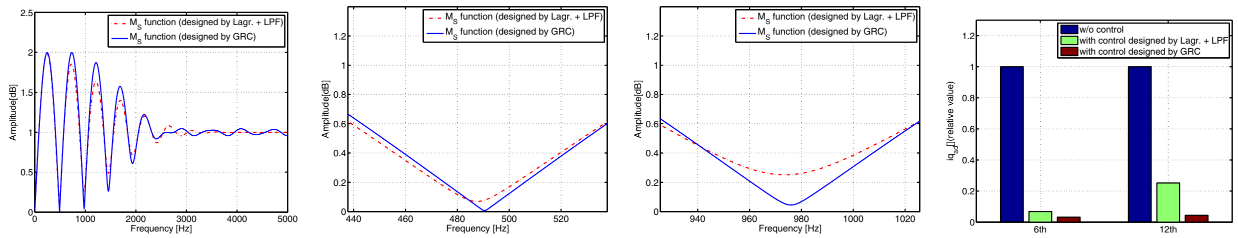
	design paramter (conventional methods)		design paramter (proposed method)
N_1	2	ε	0.05
N_2	3	ω_ε	2.5kHz
γ	2	γ_{np}	2
		Δ	0.01

TABLE II
DESIGN PARAMTER IN DESIGN 1

	design paramter (conventional methods)		design paramter (proposed method)
N_1	2	ε	0.05
N_2	3	ω_ε	3kHz
γ	2	γ_{np}	2
		Δ	0.01

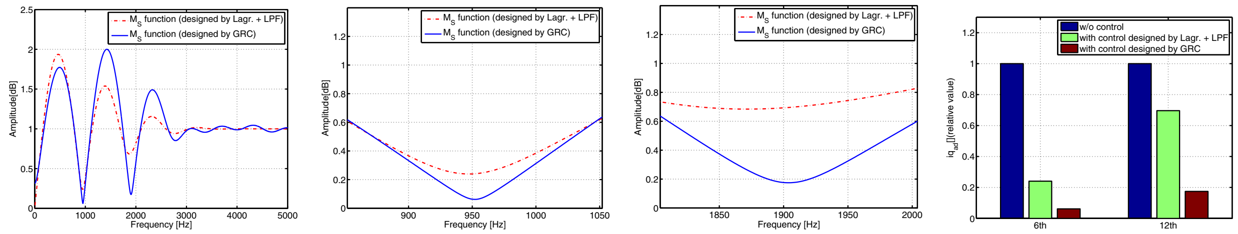
TABLE III
DESIGN PARAMTER IN DESIGN 2

value is 2. The other design parameters for conventional and proposed controllers are shown in Table II and III. Frequency characteristics of modifying sensitivity function $M_S(\omega)$ which are obtained by conventional methods and proposed methods are shown in Fig. 5 and Fig. 6. It is proven that these controllers have same cutoff frequency in Fig. 5(a) and 6(a). The differences of proposed methods and conventional methods can be seen in low frequency in Fig. 5(b) and 6(b). Furthermore, proposed controller has a large suppression ability in high frequency region. This is explained that in proposed method, LPF and Lagrange interpolation Filter can be optimized simultaneously. Note Lagrange interpolation is not a good approximation in high frequency region[14] and this can be seen in Fig. 6(c). Therefore, it is more effective to use GRC when designers need



(a) Frequency Characteristics (b) Details at 6th order frequency (c) Details at 12th order frequency (d) Box plot of 6th and 12th order disturbance rejection performance

Fig. 5. Design 1 : comparison of $M_S(z)$ between conventional and proposed methods (target frequency is 488Hz and 976Hz)



(a) Frequency Characteristics (b) Details at 6th order frequency (c) Details at 12th order frequency (d) Box plot of 6th and 12th order disturbance rejection performance

Fig. 6. Design 2 : comparison of $M_S(z)$ between conventional and proposed methods (target frequency is 952Hz and 1.91kHz)

to optimize sensitivity function at high frequency region.

V. EXPERIMENT

In this section, experimental results corresponding to the design example 1 is shown. The rotational speed is 813rpm. The controller $K_o(z)$ is obtained by discretizing the following $C(s)$ by Tustin transformation with control period T_u.

$$C(s) = \frac{Ls + R}{\tau s}, \tau = 10T_u \tag{29}$$

In the experiment, GRC is designed for q-axis current control. ZPETC needs future reference at one sample period, and it is realized by shifting the memory structure of GRC forward at one sample period. The references of dq-axis fundamental currents are set to 0A. Experimental results are shown in Fig. 7. Fig. 7(e) shows 6th and 12the component of the currents are suppressed largely with conventional and proposed methods. Proposed methods are better performance to suppress 6th and 12th components than conventional methods, which corresponds to the simulation results shown in Fig. 5(d).

VI. CONCLUSION

There are many repetitive disturbance rejection methods based on internal model principles, but few can handle multi-objective control specification. GRC is very flexible and suitable for multi objective control. In this paper, it is shown that fractional order repetitive controller can be designed easily and effectively by GRC. This method needs few iterations and designers can easily obtain the controller which meets the requirement.

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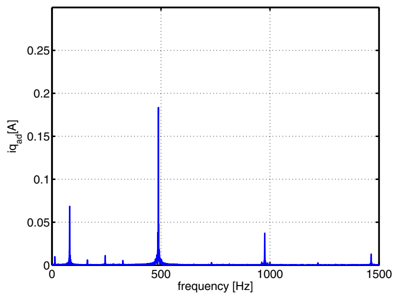
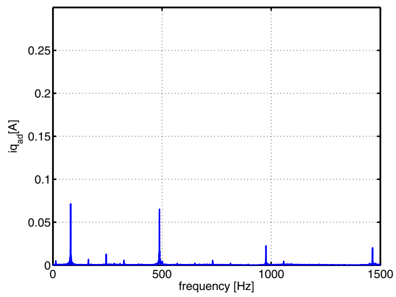
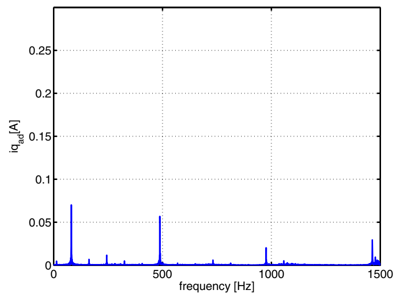


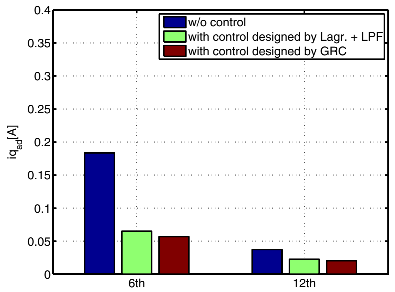
(a) FFT result of q-axis current(w/o control)



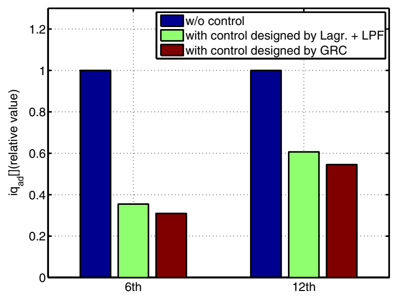
(b) FFT result of q-axis current(w/ conventional method)



(c) FFT result of q-axis current(w/ proposed method)



(d) comparizon between conventional and proposed method(real value)



(e) comparizon between conventional and proposed method(relative value)

Fig. 7. Experimental Results of fractional order repetitive control by GRC

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