Optimization of Cross Coupling Cancellation for Multiple-Receiver Wireless Power Transfer System at Changing-State

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Abstract Wireless Power Transfer (WPT) using magnetic resonant coupling is a novel technology for charging daily mobile devices. The charging system can be modeled as a multiple-receiver WPT system. Cross coupling cancellation method appending additional impedances to each receiver can remove the negative effects caused by cross coupling among multiple receivers. However, the value of additional impedances changes along with the relative position's change of charging source and mobile devices. This paper uses optimization method to obtain a fixed value for additional impedances, so that the holistic total power transfer efficiency is the largest, no matter in what arrangement the devices are.

Keyword Wireless Power Transfer, magnetic resonant coupling, cross coupling cancellation, multiple receivers, optimization.

1. Introduction

Recently Wireless Power Transfer (WPT) using Magnetic Resonant Coupling (MRC) technique has become a research hotspot [1][2]. Fig. 1 shows one scenario of WPT application, where energy is transferred from a charging panel to multiple electrical appliances, constituting a multi-receiver WPT system.

In real applications, generally multiple transmitters or multiple receivers exist simultaneously within one charging system, so the research for multi-node WPT systems is certainly needed. There are papers on multiple transmitters single receiver [3][4]. [3] worked on the layout of transmitters to get an optimal efficiency. [4] discussed about the transmission diversity. There are also papers on single transmitter multiple receivers [5]-[9]. [5] used metamaterial to build a multiple-receiver system. [6] found the optimal efficiency expression in a multi- receiver system considering the cross coupling effect. [7] designed a class-E inverter and a class-E rectifier in order to realizing actual impedance matching. [8] still worked on the efficiency optimization. [9] did research on achieving the optimal efficiency by tuning the load impedance too. Other papers are about multiple repeaters [10][11]. [10] analyzed the multiple repeaters configuration and showed the dead zones in simulation. [11] build a series of repeaters in Domino forms and found the optimal distance between each two resonators.

In multi-receiver WPT systems, the coupling among receivers is called cross coupling. Paper [12] investigated the effect caused by cross coupling in a 1-Tx 2-Rx* WPT system and proposed a Cross Coupling Cancellation (CCC) method to eliminate the effect. However, this CCC method in [12] can only be applied to fully symmetric multi-Rx WPT systems, where all receivers are same and all the mutual inductance between Tx and each Rx are same. As a solution, [13] proposed an improved method to compensate cross coupling in general multi-Rx WPT systems. Yet these two methods in [12] and [13] can only function at the resonant frequency. Paper [14] provides a CCC method to cancel the cross coupling at any frequency for multi-Rx systems.

In the system shown in Fig. 2, the cancelling capacitors have the following expressions [13].

$$
\begin{aligned}
C_{\mathrm{C1}} &= \frac{M_{\mathrm{t1}}}{M_{\mathrm{t2}}}\frac{R_2+R_{\mathrm{L2}}}{R_1+R_{\mathrm{L1}}}\frac{1}{\omega^2 M_{\mathrm{C}}}\\
C_{\mathrm{C2}} &= \frac{M_{\mathrm{t2}}}{M_{\mathrm{t1}}}\frac{R_1+R_{\mathrm{L1}}}{R_2+R_{\mathrm{L2}}}\frac{1}{\omega^2 M_{\mathrm{C}}}.
\end{aligned} \tag{1}
$$

*Tx means transmitter, Rx means receiver.

Fig. 1 Wireless charging for mobile devices.

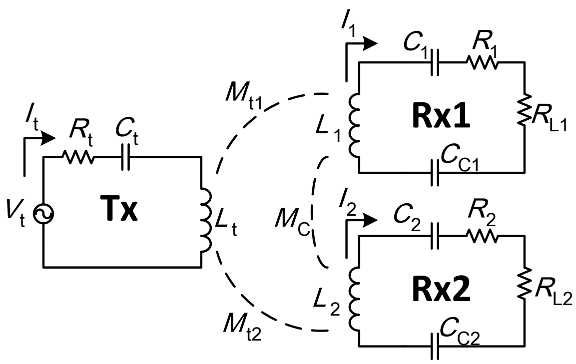

Fig. 2 1-Tx 2-Rx WPT system.

From the expressions of C_{C1} and C_{C2} we can see that C_{C1} and C_{C2} are functions of M_{t1} M_{t2} M_{C}. If any of M_{t1} M_{t2} M_{C} changes, the cancelling capacitor changes. In real applications, capacitor can be adjustable in two ways. One is using switches to choose the capacitor from many pre-positioned capacitors. In this way many capacitors are needed and they take more places, so that this method is not appropriate for mobile devices. The other one is using variable capacitors which has high costs. Both ways have obvious disadvantages.

We hope to fix the cancelling capacitors to such a specific value, that although they are not the optimal ones in most situations, the sacrifices of overall efficiency can be acceptable. In other words, a specific value of cancelling capacitor need to be found, that with this specific value, no matter how other parameters change, the overall efficiency will not decrease a lot, namely, the holistic efficiency is the largest. This problem can be called cancelling capacitor optimization problem.

2. Objective

This cancelling capacitor optimization problem turns out to be an optimization problem. A constrained optimization problem has the standard form as following [15].

$$\begin{aligned} \text{maximize} \quad & f(x) \\ \text{subject to} \quad & g(x) \geq 0 \\ & h(x) = 0 \end{aligned}$$

Where $x \in \mathbb{R}^n$, $f : \mathbb{R}^n \to \mathbb{R}$. x is called the control variable. $f(x)$ is the objective function and $g(x)$ $h(x)$ are the constraint functions.

In the cancelling capacitor optimization problem, certainly, $x = (C_{\text{C1}}, C_{\text{C2}}) \in \mathbb{R}^2$.

As to the constraint functions, although (1) denotes the expressions of cancelling capacitors, these are not the constraint functions for this optimization problem. According to (1), for each set of $(M_{\text{t1}}, M_{\text{t2}}, M_{\text{C}})$, $(C_{\text{C1}}, C_{\text{C2}})$ have different value. But in the optimization problem, $(C_{\text{C1}}, C_{\text{C2}})$ is to be fixed. $(C_{\text{C1}}, C_{\text{C2}})$ even can be less than 0, in this case inductors should be used instead of capacitors. So there is no constraint functions to this optimization problem.

The remaining element of the optimization problem is the objective function $f(x)$. This function is definitely related to the total efficiency, but it is definitely not simply the total efficiency. $f(x)$ will be illustrated in Fig. 3.

In Fig. 3, η_{max} is the maximum efficiency the system could reach. At each point of k_c, C_{C1} C_{C2} are optimized using (1), and the efficiency can therefore return to its maximum value. In this case, C_{C1} C_{C2} are always changing.

The irregular curve $\eta(k_c, C_{\text{C1}}, C_{\text{C2}})$ is the efficiency as a function of $k_c, C_{\text{C1}}, C_{\text{C2}}$. The efficiency could never be larger than the maximum efficiency.

The integration $\int \eta(k_c, C_{\text{C1}}, C_{\text{C2}})\mathrm{d}k_c$ is the area under $\eta(k_c, C_{\text{C1}}, C_{\text{C2}})$. This integration can be regarded as the holistic efficiency of the whole system at whatever situation. Since this is an integration over k_c domain, the result will be a function of $C_{\text{C1}}, C_{\text{C2}}$, namely, $f(C_{\text{C1}}, C_{\text{C2}}) = \int \eta(k_c, C_{\text{C1}}, C_{\text{C2}})\mathrm{d}k_c$.

Note that k_c is the coupling coefficient. $k_c < 1$, k_c is bounded. If k_c is too small, we can neglect it. In real application, the coupling coefficient also has a maximum value due to coils' positioning. So, $k_{c/LB} \leq k_c \leq k_{c/UB}$. The schematic diagram of $f(x)$ is in Fig. 4.

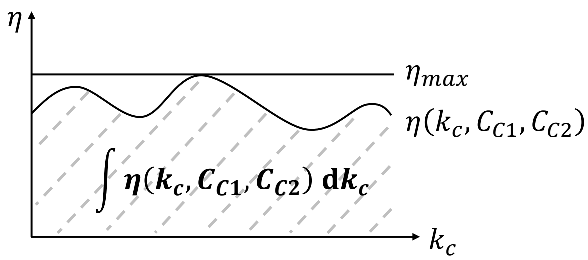

Fig. 3 Schematic diagram of $f(x)$.

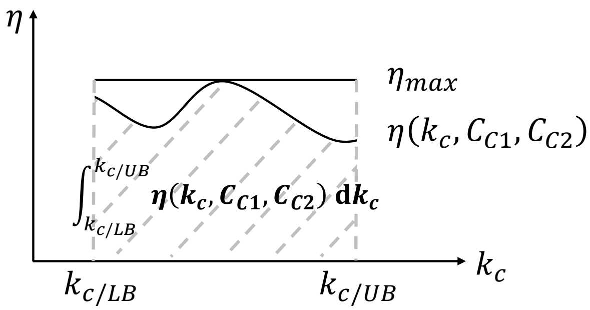

Fig. 4 Schematic diagram of $f(x)$ with boundary.

In this optimization problem, we want to find a set of (C_{C1}, C_{C2}), so that the holistic efficiency, namely, the area under $\eta(k_c, C_{C1}, C_{C2})$ is the largest. So the optimization formula turns out to be

$$\text{maximize} \qquad f(C_{C1}, C_{C2}) = \int_{k_{c/LB}}^{k_{c/UB}} \eta(k_c, C_{C1}, C_{C2})\mathrm{d}k_c. \tag{2}$$

3. Solution – A Case Study

Because many factors are depended on case, a case study has been executed as the solution. There are some preconditions of this case study. Firstly, a 1-Tx 2-Rx multi-Rx system which is shown in Fig. 2 has been used, with its parameters shown in Table 1. Secondly, the load resistances are optimized and constant. Thirdly, assuming the application being the charging panel in Fig. 1, only M_C changes with the positions of mobile devices change. Fourthly, assume k_c varying from 0.05 to 0.35.

The three steps to solve the optimization problem (2) are, calculating the efficiency as a function of k_c, C_{C1}, C_{C2}, deducing the integration of it, and obtaining the maximum point.

The efficiency function is extreme long but has a form shown in (3). Its numerator and denominator are polynomials of

TABLE I
Parameters used in case study.

	Tx	Rx1	Rx2
R(Ω)	0.1	0.1	0.3
L(H)	4μ	4μ	8μ
C(F)	1μ	1μ	0.5μ
R_L(Ω)		0.356	1.068

k_{t1}	0.15
k_{t2}	0.1
k_c	0.05$\sim$0.35

k_c. C_{C1}, C_{C2} lie in the coefficients $a_n, n = 0, 1, 2$ and $b_m, m = 0, 1, 2, 4$.

$$\eta(k_c, C_{C1}, C_{C2}) = \frac{a_2 k_c^2 + a_1 k_c + a_0}{b_4 k_c^4 + b_2 k_c^2 + b_1 k_c + b_0} \tag{3}$$

Equation (3) is a rational function. There is no analytical expression for the integration of (3). A general way to calculate the integration of a rational function is through factorization [16], but this way cannot be used in obtaining this integration.

Assume the general form of a rational function is

$$\frac{P(x)}{Q(x)} = \frac{a_0 x^n + a_1 x^{n-1} + \cdots + a_n}{b_0 x^m + b_1 x^{m-1} + \cdots + b_m}. \tag{4}$$

It could be factorized into

$$\begin{aligned} \frac{P(x)}{Q(x)} =& \frac{A_1}{(x-a)^\alpha} + \frac{A_2}{(x-a)^{\alpha-1}} + \cdots + \frac{A_\alpha}{(x-a)} + \\ & \frac{B_1}{(x-b)^\beta} + \frac{B_2}{(x-b)^{\beta-1}} + \cdots + \frac{B_\beta}{(x-b)} + \\ & \frac{M_1 x + N_1}{(x^2+px+q)^\lambda} + \cdots + \frac{M_\lambda x + N_\lambda}{(x^2+px+q)} + \\ & \frac{R_1 x + S_1}{(x^2+rx+s)^\mu} + \cdots + \frac{R_\mu x + S_\mu}{(x^2+rx+s)} \end{aligned} \tag{5}$$

The integration of form (5) is easily to calculate. However, all coefficients in (4) must be known numbers. Since the coefficients in (3) contain C_{C1} and C_{C2}, (3) cannot be factorized into form (5), namely the integration of (3) cannot be calculated.

Recall the geometrical definition of integration. Integration of a curve is the area between the curve and horizontal axis, as shown in Fig. 3 and Fig. 4. So we could calculate the sum of lots of small rectangles to obtain the approximate integration. An example is shown in Fig. 5. The expression of integration will be reformed into

$$f(C_{C1}, C_{C2}) = \int \eta(k_c)\mathrm{d}k_c \approx \sum \eta(k_c)\Delta k_c. \tag{6}$$

By this mean the integration could be calculated as a function of (C_{C1}, C_{C2}), with some errors.

To look for the maximum point of a function, we could use some computer softwares, for example, Mathematica. There is

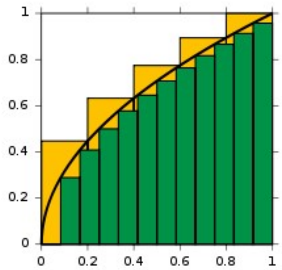

Fig. 5 Example: Approximations to integral of $\sqrt{x}$ from 0 to 1. [17]

TABLE II
PARAMETERS USED IN FIG. 7.

	η_1	η_2
$C_{\text{C1}}(F)$	10.607μ	52.378μ
$C_{\text{C2}}(F)$	31.820μ	1.571μ

a function called Maximize in Mathematica, which is perfectly suitable for this problem. With an assigned approximate maximum point, Mathematica returns the precise one, which are $C_{C1} = 13.345\mu, C_{C2} = 0.91507\mu$.

Using the calculated values of $C_{\text{C1}}, C_{\text{C2}}$, the efficiency curve versus k_c has been plotted, shown in Fig. 6. Fig. 6 agrees with the hypothesis in Fig. 4.

Using other values of $(C_{\text{C1}}, C_{\text{C2}})$, some curves have been plotted in Fig. 7, from which we could see, the area under η_opt is the largest as expected. The exact values of $(C_{\text{C1}}, C_{\text{C2}})$ is shown in Table 2.

Fig. 8 contains the efficiency curve without using any cancelling capacitors $C_{\text{C1}}, C_{\text{C2}}$. We can see that the efficiency without $C_{\text{C1}}, C_{\text{C2}}$ decreases with k_c increases. At most points of k_c, η_opt is larger than $\eta\_without$, namely, the optimization method successfully increase the efficiency with fixed cancelling capacitors.

4. CONCLUSION

This paper successfully applies the optimization method to Cross Coupling Cancelling problem at changing-state. A case study has been executed to prove the performance of optimization. The analytical expressions of optimal value of cancelling capacitors cannot be deduced. Errors always exist due to the computational accuracy. Real applications needs to be further considered.

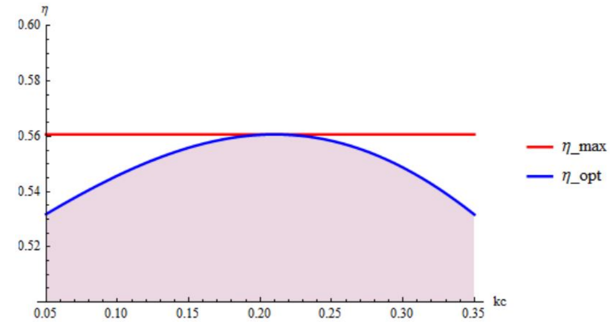

Fig. 6 Verification of results.

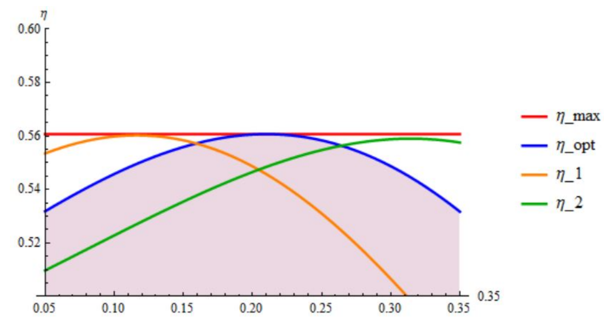

Fig. 7 Comparison of different $(C_{\text{C1}}, C_{\text{C2}})$.

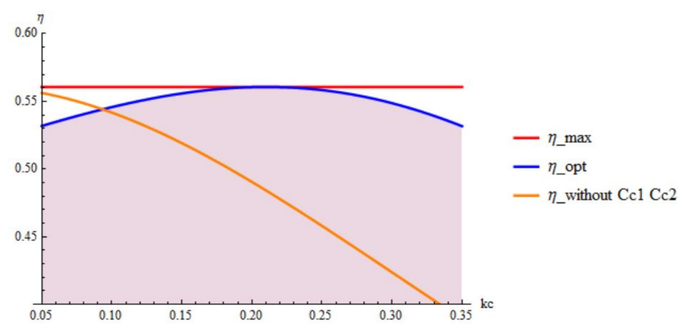

Fig. 8 Efficiency without using $C_{\text{C1}}, C_{\text{C2}}$.

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