# Study on an Extension of Generalized Repetitive Control to Variable Speed Region Based on Oppenheim's Frequency Transformation

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## 1 Introduction

Generalized Repetitive Control(GRC) proposed in [1] has an excellent performance to reject periodic disturbances. This controller is designed by convex optimization, and it makes the difficulty to re-design GRC online. In motor drive applications, disturbance frequency changes according to the rotation speed. Therefore, it is needed to re-design the GRC controller online. In this paper, we propose a new algorithm which enables us to re-design GRC online. This method is based on FIR re-designing method proposed in [2].

## 2 FIR Redesigning Method

When GRC is added to the loop, the frequency characteristics of the modifying sensitivity function is expressed as

$$M_S(z) = 1 - X(z) \tag{1}$$

$$X(z) := \sum_{k=K_1}^{K_2} X_k z^{-k}$$
(2)

This research objective is to transform the frequency characteristics of the modifying sensitivity function by the transformation of the frequency characteristics of X(z). X(z)is designed as a linear phase FIR filter to which FIR redesigning method proposed in [2] is applied.

Firstly, X(z) is expressed in the form

$$X_{0}(z) = z^{-N_{d}} \sum_{k=-N}^{N} a_{k} z^{k}$$
  
:=  $H_{0:p}(z) H_{0:a}(z)$  (3)

$$H_{0:p}(z) := z^{-N_d} \tag{4}$$

$$H_{0:a}(z) := \sum_{n=0}^{N} A_n \left(\frac{z+z^{-1}}{2}\right)^n$$
(5)

The amplitude of  $H_{0:p}(z)$  is 1 and there is no phase delay in  $H_{0:a}$ . The characteristics of  $H_{0:p}(z)$  and  $H_{0:a}(z)$  is designed separately to achieve desirable frequency characteristics. The phase characteristics are stretched by adding delay operators to  $H_{0:p}(z)$ . Noting that the amplitude characteristics of  $H_{0:a}$  is a polynomial function of  $\cos \Omega$ , the amplitude characteristics is transformed by distorting the fre-

(a) amplitude (b) phase

Figure 1: Simulation Result of FIR Redesigning Method

quency axis. Such a transformation is realized by the substitution of variables

$$\frac{z+z^{-1}}{2} = K_0 + K_1 \left(\frac{Z+Z^{-1}}{2}\right) \tag{6}$$

The property of this linear mapping is decided by the parameters  $K_0$  and  $K_1$ . These parameters should be set according to the relationship between prototype frequency response  $H_{1:a}(z)$  and the transformed frequency response  $H_{1:a}(z)$ . In this paper, the parameters which equalize the properties of  $H_{1:a}(z)$  and  $H_{0:a}(z)$  at zero and cutoff frequency are selected. Figure 1 demonstrates this re-designing method.

#### **3** Conclusion

In this paper, a new algorithm which enables to re-design GRC online is proposed. It is our next step to update the filter coefficients dynamically to achieve dynamic loop shaping.

### References

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