Abstract—This paper generalizes the structure of Force-Sensor-less Power Assist Control (FSPAC) which is a essential technology to design a feedback controller in a human-friendly way. Based on the analysis of the control structure, this paper clarifies the functions of the generalized force-sensor-less power assist control; particularly the functions of two force observers in FSPAC are made clear. Since the FSPAC consists of a positive feedback control, the robustness of the FSPAC is much weaker than the other power assist control with force sensors. This paper clarifies the trade-off between assistance performance and weak robustness in the FSPAC. Comparing power assist control derived from model following design and the disturbance observer based design, the limitation of assistance performance and methods to adjust the gain are revealed. Finally methodology of design the feedback gain in the FSPAC is discussed.

I. INTRODUCTION

There are many researches and applications which uses power assistance system. Power steering system in vehicle operation has been a typical application of power assistance. Besides that conventional application, more daily life application such as power assist wheelchairs [1], [2] and wearable electro-mechanical suits [3] are highlighted recently as new applications of power assistance system. As the numbers of these power assistance applications increases, design methodology of the control part of that power assistance needs to be investigated.

Since the power assist control needs to measure the force to assist, force sensors such as strain sensors are used in power assistance system. However, there are researches on the power assist controls which do not use any force sensor and estimate the force using just encoders [4], [5]. They use the reaction force observer methodology which is based on the disturbance observer technology [6] to estimate the force and have obtained successful experimental results in their researches.

However, the reaction force observer has some disadvantage that it requests the precise dynamic model and friction force values. To overcome this disadvantage, the dynamics learning methodology is developed to adapt the parameters in dynamic model to the precise dynamics [7] and even more there is model independent force observer approaches [9].

Although there are some differences in these researches, most of them use input and output signal of the plant and plant dynamics for the estimation of the force. These force estimation have some advantage over the force measurement; the estimation in the FSPAC is fast while the delay in the measurement of force sensors is fairly long, which means the force control using the estimation can make faster response and it can be free from sensor noise. In spite of these advantages, the FSPAC has significant robustness problem against the model changes [4], [11]. Considering these conditions, this paper reveals the limit of robustness of FSPAC and design methodology of the FSPAC to adjust weak robustness. Note that the FSPAC discussed in this paper does not adopt any adaptive or learning controller but just is a time invariant linear controller.

The paper is organized as follows: In Section II, FSPAC is divided into four control sections; the framework to analyze the FSPAC is proposed in Section II. Section III analyzes the function of each section in FSPAC and compares the proposed generalized FSPAC structure with other structures that can be thought of as alternative FSPAC structure. In Section IV, the weakness in robustness of the FSPAC is investigated and design method of the gains in FSPAC is discussed. Section V concludes this paper.

II. GENERALIZED STRUCTURE OF THE FORCE-SENSOR-LESS POWER ASSIST CONTROL

The structure force-sensor-less power assists control is generalized as Figure 1; two disturbance observers, model impedance, and feedback gain. Inner disturbance observer is the conventional disturbance observer which aims to reject all the external force. The outer disturbance observer which is usually called a reaction force observer or force observer is for the estimation of the force to assist.

The model impedance decides the extent of power assistance. Since the feedback controller in FSPAC controls the system output $y_c$ to track the output $y_{imp}$ of this model impedance, this model decides the impedance of the controlled system against external force. If the impedance in the model is smaller than that of the original plant which means $J_M$ and $B_M$ are set to $J_M < J, B_M < B$, the model impedance achieves power assistance. This is how the FSPAC realizes power assistance.

The feedback controller which produces the control input proportional to the error determines the tracking characteristics. The controller consists of the conventional PID controller. The higher this gain is, the better the tracking performance will be. As will be discussed later, this feedback controller
determines the robustness - the gain margin of the system, too.

The structure of Figure 1 has been used as the fundamental structure of the FSPAC in many researches [2], [4]. This paper divides the controller into four sections so that the functions and design of the FSPAC can be dealt with based in terms of each section. This also enables us to explain why the structure of the FSPAC should be like Figure 1 and to compare with other considerable FSPAC structures.

A. Estimation of External Force to Assist

Although there is a research [1] which adopted only one disturbance observer to estimate the force to assist, and omitted the inner disturbance observer, two disturbance observers are usually adopted [2], [4]. This may seem to have a functional conflict - rejection and amplification of external force - between the functions of these two disturbances. This point will be made clear in Section III-A.

In both cases, a disturbance observer estimates the external force as a force observer. In force observer, differently from the inner disturbance observer, some pre-measured disturbance information such as Coulomb friction is subtracted from the estimated force as they are not force to be assisted. This kind of force estimation is called Reaction force observer [4].

Methodologies other than a linear disturbance observer can substitute for the linear disturbance observer ([7] to [10]). They utilize a known model dynamics and system output and calculate the external force to the system based on them. The accuracy of the dynamic model determines the quality of the estimated force; if there is some modeling error in dynamic model or unknown dynamics, it will affect the estimation accuracy and result in inaccurate force estimation.

To be free from this modeling error, some learning methodologies are adopted [7], [8]. In [9], Neural Network learning is adopted and in [10], sliding mode approach is adopted to estimate the force. It is true that these researches can be substitutes of the linear disturbance observer, but this paper adopts the linear system analysis so that the robustness can be analyzed in terms of linear system analysis.

B. Feedback Control to Design Assistance Characteristics

Power assist control is different from the force control which is focusing on the tracking of the force value. In the power assist control, the output behaviors such as position or velocity of the target plant are more important states to be controlled [1].

For this end, power assist controller often takes the form of compliance control [2] which is illustrated in Figure 2. It is an important idea in the analysis of power assist control that feedback controller in the velocity control form [1] can be a unified form of position, velocity, and force feedback controllers and also include the complican control form of Figure 2.

Figure 3 is the equivalent velocity control of the compliance control in Figure 2. This point shows that the division of model impedance and feedback controller in Figure 1 can also describe power assist control in the compliance control form.

Based on these considerations on force estimation and generalized feedback control, the analysis on FSPAC is conducted in the following sections.

III. ANALYSIS OF THE GENERALIZED FSPAC

For simple and general description, the control blocks in Figure 1 is characterized like Figure 4. Q filters in two
disturbance observers are illustrated as \( Q_i \) and \( Q_o \), the inverse dynamics model is described as \( P_n^{-1} \), the model impedance is described as \( P_M \), \( P \) is the real plant, and the feedback controller is described as \( A \). This notation is used in the following analysis.

The inner disturbance observer can improve model matching performance of the FSPAC. The second term \(-PQ_i\) can eliminate the effect of the first term \( P \), if \( Q_i \approx 1 \). This is conventional usage of the disturbance observer; this elimination of the effect of the path (1) can help the controlled plant to follow the model impedance \( P_M \).

Besides this model matching characteristics, two observers can distinguish the disturbances to be assisted and to be rejected by setting different processing between the two observers.

### B. Decision of Parameters

With high gain \( A \) on the condition of \( Q_i \approx 1 \) and under the assumption \( P_n \approx P \), the transfer function becomes

\[
T_{cl} = \frac{PQ_oP_M}{P + Q_oP_M(1 - P_n^{-1}P)} \approx Q_oP_M. \tag{2}
\]

This illustrates that \( Q_o \) determines the frequency bandwidth where the model impedance is realized. In the frequency band where \( Q_o \approx 1 \), the transfer function from the external force to the system output will be \( P_M \), and in the frequency band where \( Q_o \approx 0 \), the impedance will be 0 and the force will not be assisted.

The idea how to design \( Q_i \) and \( Q_o \) are made clear; \( Q_i \) determines to what frequency the undesirable external force should be rejected and model matching characteristics should be kept. \( Q_o \) determines the frequency bandwidth where the assistance is effective. Usually, these two time constants are set as same value.

### C. Comparison with Other Force-sensor-less Power Assist Control

FSPAC structure are described in Figure 6 and 7 can be considered as substitute structure of the FSPAC.

Figure 6 has only one disturbance observer as a force observer. This structure is motivated from the concern of the conflict between two disturbance observers. With \( Q_i = 0 \) in Equation (1), the transfer function can be calculated as

\[
T_{cl} = \frac{P(1 + Q_oAP_M)}{1 + A(P + Q_oP_M(1 - P_n^{-1}P))}. \tag{3}
\]

Since the second \( Q_i \) term in Equation (1) is removed, the first term \( P \) remains in the numerator and affects the output, which may result in poor model matching performance. However, the controlled plant does not necessarily need to match the model impedance to achieve power assistance. Even though the first term \( P \) is undesired, the effect of the external force, it does not deteriorate assistance performance but merely worsen model matching performance. Additionally, regardless of the existence of the inner disturbance observer, if \( A \gg 1 \), the last term \( PQ_oAP_M \) plays dominant role in the system and the first term \( P \)'s effect decreases to negligible level.

By removing the inner disturbance observer, the structure is simplified and consequently becomes more straightforward.

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**A. Functions of two Force Observers**

![Fig. 4. Analysis of FSPAC Structure](image)

![Fig. 5. Paths of External Force under FSPAC](image)
without deteriorating power assistance performance. One disadvantage is that it cannot reject undesirable external force. Although this structure also can cut out the undesirable external force in the control loop by subtracting the undesirable force from the estimated force, it does not reject it actively unlike the FSPAC with the inner disturbance observer.

Another possible structure of FSPAC is illustrated in Figure 7. The difference of this structure is that the control input which is used as an input signal in the force observer is different from that for the inner disturbance observer. The input to the force observer is located outside the disturbance observer.

This structure is motivated by the idea that the inner disturbance observer can nominalize the plant making the system more robust against the modeling error. It seems that if the inner disturbance observer is built inside the outer force observer, the plant which is controlled by the feedback control of the disturbance observer will be nominalized and the estimation of the external force by the force observer can be more robust to the modeling error.

The analysis proves this idea is mistaken. The transfer function under this control described in Equation (4) explains this fact.

\[ T_{cl3} = \frac{P(1 - Q_i + (Q_i - 1)Q_oAP_M)}{1 + Q_i(P_n^{-1}P - 1) + A(P + Q_oP_M(Q_i - 1)(1 - P_n^{-1}P))} \]  

(4)

Compared with Equation (1), \( Q_i - 1 \) is multiplied by the \( Q_oAP_M \) term. The last term in the denominator explains the apparent robustness. If \( Q_i \) becomes 1, the effect of modeling error \( (1 - P_n^{-1}P) \) becomes unable to alter the system.

However, the term \( (Q_i - 1) \) also changes the effect of \( Q_oAP_M \) which is related with power assistance performance. Note that the last term of the numerator will be zero if \( Q_i \) becomes 1 making the whole numerator zero. This explains, with the unity \( Q_i \), all the external force will be rejected by this feedback structure and it fails to assist the external force.

This failure is originated from inability to estimate external force; since the inner disturbance observer eliminates the effect of external force to system output, no deviation is caused by the external force in system output, which makes the estimation of external force using the outer external observer impossible.

Based on these analysis, it is concluded that the structure in Figure 7, though it seems to have robustness to the modeling error, it cannot achieve power assistance.

IV. ROBUSTNESS LIMITATION OF FSPAC

A. FSPAC as a Model Following Control Design

Let us discuss the design of a force-sensor-less controller using the structure in Figure 8. The plant is \( \frac{1}{J_s + B} \) and we want to control it so that the output \( y \) moves as if it has the impedance characteristic of \( \frac{1}{J_M + \Delta B M} \) against the external force \( f \) by the feedback controller \( C(s) \).

\[ C(s) \rightarrow f \rightarrow \frac{1}{J_s + B} \rightarrow y \]

(8)

Under this problem formulation, the controller \( C(s) \) can be designed based on the model following control design. By substituting the general plant \( P(s) \) for \( \frac{1}{J_s + B} \), the formulation can be generalized. The answer is

\[ C(s) = (J_M - J_n)s + (B_M - B_n) = P_M^{-1}(s) - P_n^{-1}(s), \]  

(5)

where \( J_n, B_n \) and \( P_n \) are the nominal parameters and plant for the real plant.

If this model following control is designed for power assistance without force sensors, the model impedance should be smaller than that of real plant, that is, \( J_M < J_n \) and \( B_M < B_n \). This makes the feedback controller positive feedback and results in weak robustness caused by small stability margin.

B. Limitation of Robustness

Since it is positive feedback system, the open loop frequency characteristics starts at the phase of \( 180^\circ \), which explains why it has small gain margin. In the case of Figure 8 with the feedback controller of Equation (5), the gain margin is given as

\[ \left| 1 + \frac{B_M - B_n}{B} \right| = \left| 1 - \frac{1}{\Delta B} + \frac{B_M}{B} \right|, \]  

(6)
where the modeling error in $B$ is described as

$$B = B_n\Delta_B.$$  \hspace{1cm} (7)

From this gain margin calculation, we can tell
- Larger $B_M$ (but less than $B$) or small $B$ guarantees larger stability margin. This, however, conflicts with the assistance performance.
- If the nominal model $B_n$ is set larger than real plant $B(\Delta_B < 1)$, the gain margin will decrease. In the opposite case where $\Delta_B > 1$, the margin will increase.

C. Adjustment of Gain Margin

In the case of the FSPAC in Figure 4, the gain margin is given as Equation (8) under the assumption $Q_i \simeq 1$ and $Q_o \simeq 1$.

$$\left|1 + \frac{B_nB_M + A(B_M - B_n)}{BA}\right| = \left|1 + \frac{B_M}{A\Delta_B} - \frac{1}{\Delta_B} + \frac{B_M}{B}\right|$$  \hspace{1cm} (8)

A tunable variable $A$ is included in the margin. If $A$ is small, the second term $\frac{B_M}{A\Delta_B}$ can produce appreciable additional gain margin, while the small gain $A$ deteriorates the tracking performance and thus decreasing assistance.

Large gain to obtain sufficient assistance, will decrease the effect of the second term leading to the same gain margin with Equation (6). This analysis clarifies that compared with the simple model following control design, the proposed generalized FSPAC design can control the trade off between the assistance performance and the robustness with the feedback gain.

Using the feedback control form of the FSPAC in Figure 9, the bode diagram is illustrated in Figure 10.

This bode diagram describes that the gain margin is more significant than the phase margin; since the phase of the FSPAC is starts from 180°, and the gain is not so high, the phase has enough stability margin, while the gain has small margin.

To investigate the effect of the control parameters on the robustness, the gain margin is drawn changing the parameters $A, Q_i$ and $Q_o$. To simplify the discussion, two time constants of two observers $Q_i$ and $Q_o$ are set same, which is also a practical way of design. Figure 11 is the gain margin with regard to the feedback gain $A$ and the time constant of two observers. This figure explains that feedback gain is strongly related with the gain margin and thus robustness[12].

Figure 10 and Figure 11 and discussions above assures that our analysis of robustness using the gain margin and the feedback gain is quite effective.

\[\text{Bode Diagram}
\]

**Fig. 10.** Gain Margin and Phase Margin of FSPAC

**Fig. 11.** Gain Margin with FB Gain and Time Constant Changes

D. Weak Robustness in Phase Assist Control

Another interesting discussion on the assistance control is phase assist control. While the power assist control focuses on the amplification of the power amplitude, the phase assist control focuses on the response speed of the plant and changes the time constant of it.

To investigate the characteristics of this phase assist control, two bode diagrams of FSPAC are described in Figure 12 and Figure 13.

In Figure 12, the model impedance is set to 0, and in Figure 13, $P_M$ is set to 0, and $P_M$ is set to 0.8. The largest difference between these two model impedances is the ratio of the model time response (8) to the original time response(8). When this ratio is smaller than the unity, the response of the output to the external force will be faster than that of the original system under the force-sensor-less power assist control.

This is the concept of the phase assist control. The ratio of Figure 12 is set to 0.8 while that of Figure 13 is set to 1.1, which means Figure 12 is the frequency characteristics of the phase assist control while Figure 13 is not.
This equation suggests that design. replaces the Q filter can shed some insight into the FSPAC when we design FSPAC with more detailed model impedance. Then the open loop characteristic is simplified to

\[ Q_o = \frac{1}{D_o} \cdot P_M = \frac{1}{D_M}, \quad P_n = \frac{1}{D_n}, \quad P = \frac{1}{D}, \quad Q_i \simeq 1. \]  

(9)

Then the open loop characteristic is simplified to

\[ T_{ol} = \frac{A(D_M D_o - D_n)}{(D_M D_o + A)D} \simeq \frac{D_M D_o - D_n}{D} \quad \text{(when } A \gg 1). \]  

(10)

This equation suggests that \( D_M D_o - D_n \) is significant equation when we design FSPAC with more detailed model impedance \( P_M \).

Another criterion of FSPAC design is \( Q_i - AQ_o P_M \) in Figure 14 that redescribes the structure in Figure 4 in the form of the conventional disturbance observer. \( Q_i - AQ_o P_M \) which replaces the Q filter can shed some insight into the FSPAC design.

V. CONCLUSION

This paper proposes the general form of force-sensor-less power assist control structure and compares with it other possible structures concluding the proposed general form has some advantage over others.

Also this paper clarifies the limit of robustness of FSPAC in terms of gain margin and shows how the FSPAC changes the margin. A new concept of phase assist control is proposed and difficult in guaranteeing the robustness in the phase assist control is explained.

Some criterions are also proposed in order to develop the design discussing these problems in FSPAC. More detailed discussion on methodologies to use these criterions particularly from the viewpoint of the phase should be researched as future work.

REFERENCES