Tracking control method for high-precision stage with continuous time unstable zeros

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Background

Zeros in transfer function

- Continuous time transfer function

$$P_c(s) = \frac{-(s-140)(s+100)}{s(s+2000)(s+2)(s^2+20s+40000)}$$

□ : Unstable zeros in continuous tf

- Discretized transfer function

Intrinsic zeros — Discretization zeros

$$P_s[z_s] = \frac{K(z_s-1.014)(z_s-0.9900)(z_s+3.547)(z_s+0.2543)}{(z_s-1)(z_s-0.9998)(z_s-0.8187)(z_s^2-1.998z_s+0.998)}$$

□ : Unstable zeros in discrete tf $\quad z_s = e^{sT_u},\ T_u = 100\ \mu s$

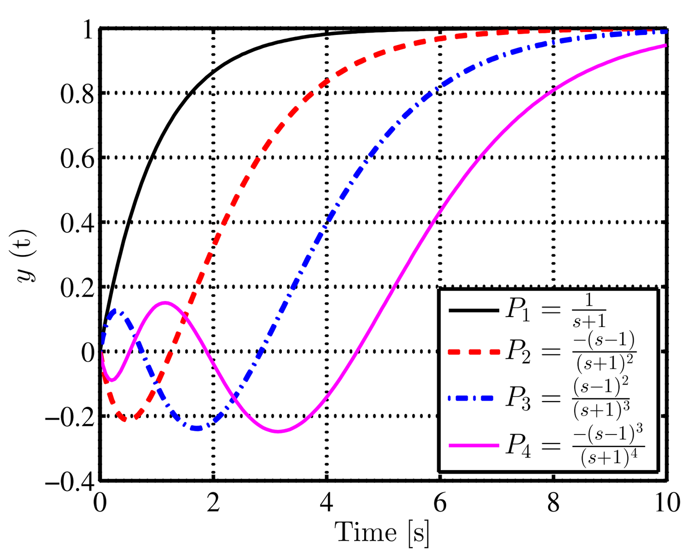
- Intrinsic zeros
  have counterpart in $P_c(s)$ and
  are generated from sensor and actuator collocation.
- Discretization zeros
  are generated by discretization and
  are approximated by Euler-Frobenius polynomial.

[K. Åström, *et al.*, 1984]
[T. Hagiwara, *et al.*, 1993]

Relative order	Discretization zeros
2	−1
3	$-2-\sqrt{3}, (-2-\sqrt{3})^{-1}$ (≈ −3.7, −0.26)

Unstable zeros problem

- Unstable poles in inversion system
- Undershoot in step response
- Example: Hard disk drive, robot, high-precision stage, boost converter, airplane…



Method

Plant definition

- Realized by control canonical form

$$P_c(s) = \frac{B(s)}{A(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_0}{s^n + a_{n-1}s^{n-1}\cdots + a_0}$$

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_c\boldsymbol{x}(t) + \boldsymbol{b}_c u(t),\ \ y(t) = \boldsymbol{c}_c\boldsymbol{x}(t)$$

$$\boldsymbol{x}[k+1] = \boldsymbol{A}_s\boldsymbol{x}[k] + \boldsymbol{b}_s u[k],\ \ y[k] = \boldsymbol{c}_s\boldsymbol{x}[k]$$

State trajectory generation

- Stable inversion for continuous unstable zeros

1. Stable & unstable decomposition

$$B(s)^{-1} = F^{\mathrm{st}}(s) + F^{\mathrm{ust}}(s)$$

$$f^{\mathrm{st}}(t) = \bar{\mathcal{L}}^{-1}\left[F^{\mathrm{st}}(s)\right], \bar{f}^{\mathrm{ust}}(t) = \bar{\mathcal{L}}^{-1}\left[F^{\mathrm{ust}}(-s)\right]$$

2. State trajectory generation for stable part

$$\boldsymbol{x}_d^{\mathrm{st}}(t) = \begin{bmatrix} x_{1d}^{\mathrm{st}}(t) & x_{2d}^{\mathrm{st}}(t) & \cdots & x_{nd}^{\mathrm{st}}(t)\end{bmatrix}^{\mathrm{T}} = \int_{-\infty}^{t} f^{\mathrm{st}}(t-\tau)\boldsymbol{r}(\tau)d\tau$$

3. State trajectory generation for unstable part

$$\boldsymbol{x}_d^{\mathrm{ust}}(t) = \begin{bmatrix} x_{1d}^{\mathrm{ust}}(t) & x_{2d}^{\mathrm{ust}}(t) & \cdots & x_{nd}^{\mathrm{ust}}(t)\end{bmatrix}^{\mathrm{T}} = \left.\int_{-\infty}^{\bar{t}} \bar{f}^{\mathrm{ust}}(\bar{t}-\bar{\tau})\boldsymbol{r}(-\bar{\tau})d\bar{\tau}\right|_{\bar{t}=-t}$$

Convolution between time axis reversed reference and imaginary axis reversed unstable zeros, and then time axis reversed again

4. Overall state trajectory

$$\boldsymbol{x}_d(t) = \boldsymbol{x}_d^{st}(t) + \boldsymbol{x}_d^{ust}(t)$$

Multirate feedforward [Fujimoto, Hori, Kawamura, 2001]

- Stable inversion for discretization zeros

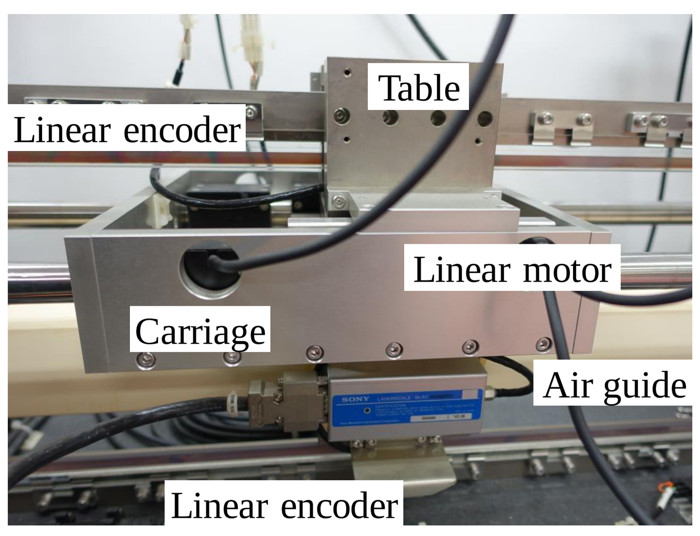
Singlerate system $\boldsymbol{x}[k+1] = \boldsymbol{A}_s\boldsymbol{x}[k] + \boldsymbol{b}_s u[k],\ \ y[k] = \boldsymbol{c}_s\boldsymbol{x}[k]$

Multirate sytem $\boldsymbol{x}[i+1] = \boldsymbol{A}\boldsymbol{x}[i] + \boldsymbol{B}\boldsymbol{u}[i],\ \ y[i] = \boldsymbol{c}\boldsymbol{x}[i]$

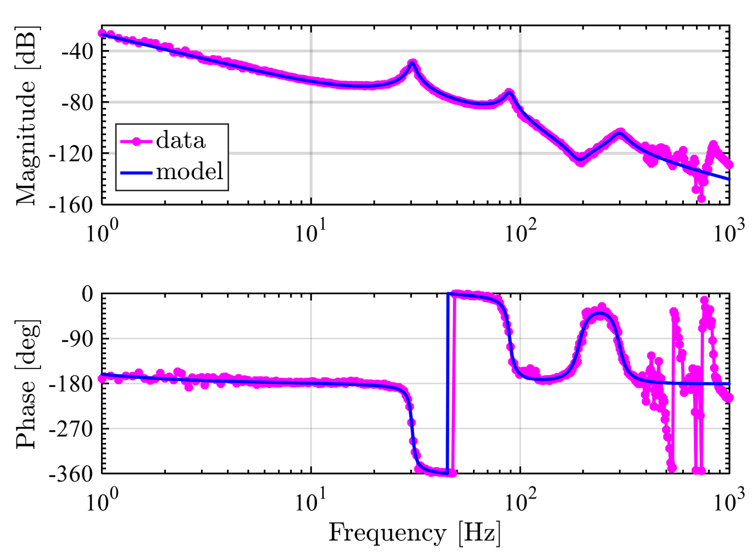
$$\boldsymbol{A} = \boldsymbol{A}_s^n,\ \ \boldsymbol{B} = \begin{bmatrix}\boldsymbol{A}_s^{n-1}\boldsymbol{b}_s & \boldsymbol{A}_s^{n-2}\boldsymbol{b}_s & \cdots & \boldsymbol{A}_s\boldsymbol{b}_s & \boldsymbol{b}_s\end{bmatrix}, \boldsymbol{c} = \boldsymbol{c}_s$$

$$\boldsymbol{u}_o[i] = \boldsymbol{B}^{-1}(\boldsymbol{I} - z^{-1}\boldsymbol{A})\boldsymbol{x}_d[i+1]$$

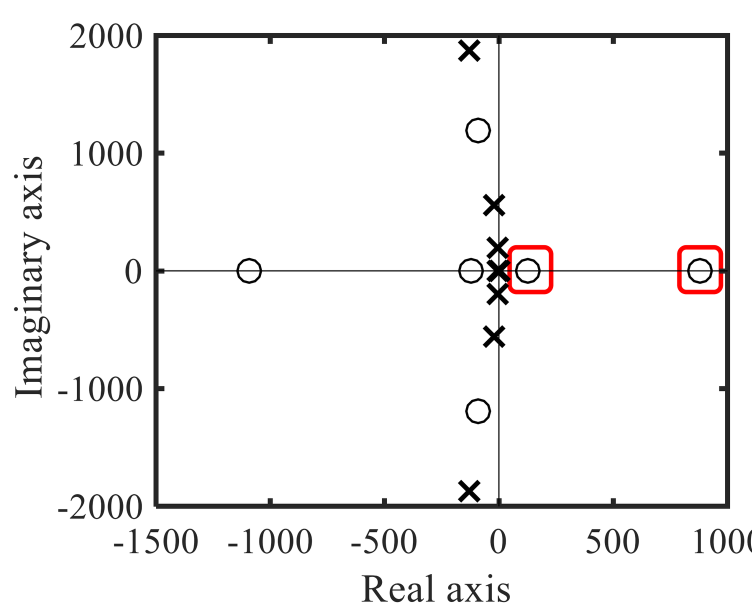
Experiments



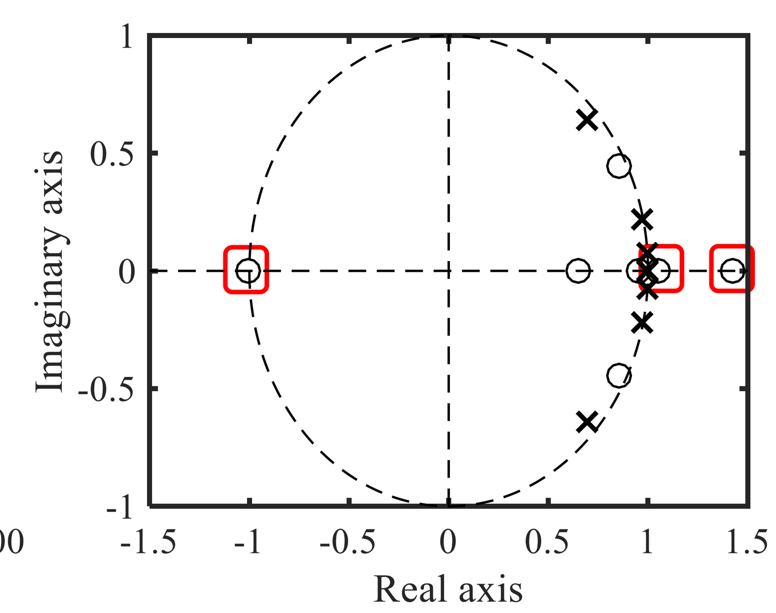
(a) High-precision stage



(b) FRF and 8th order model

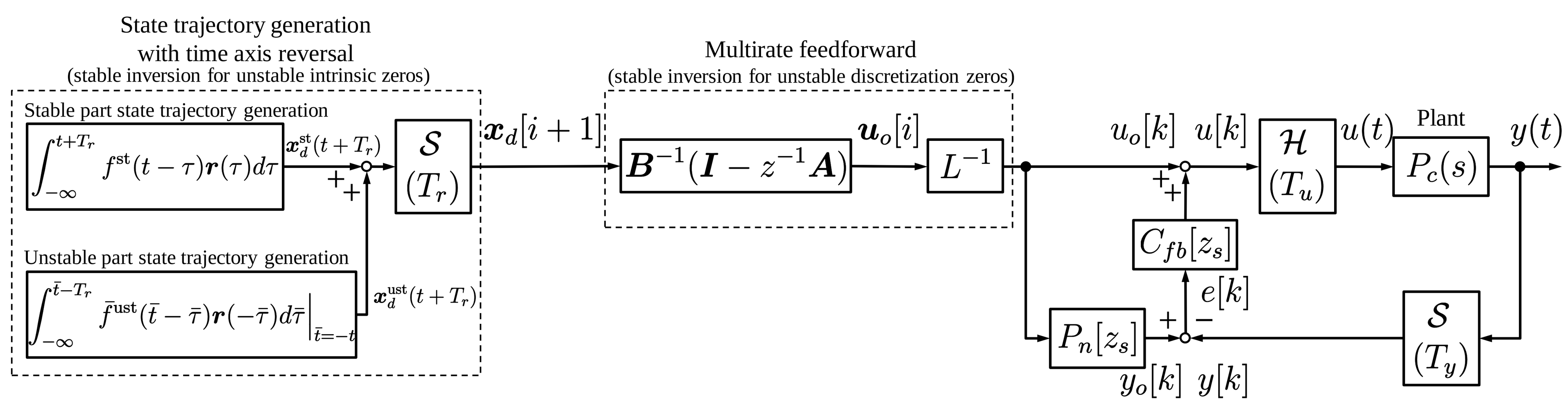


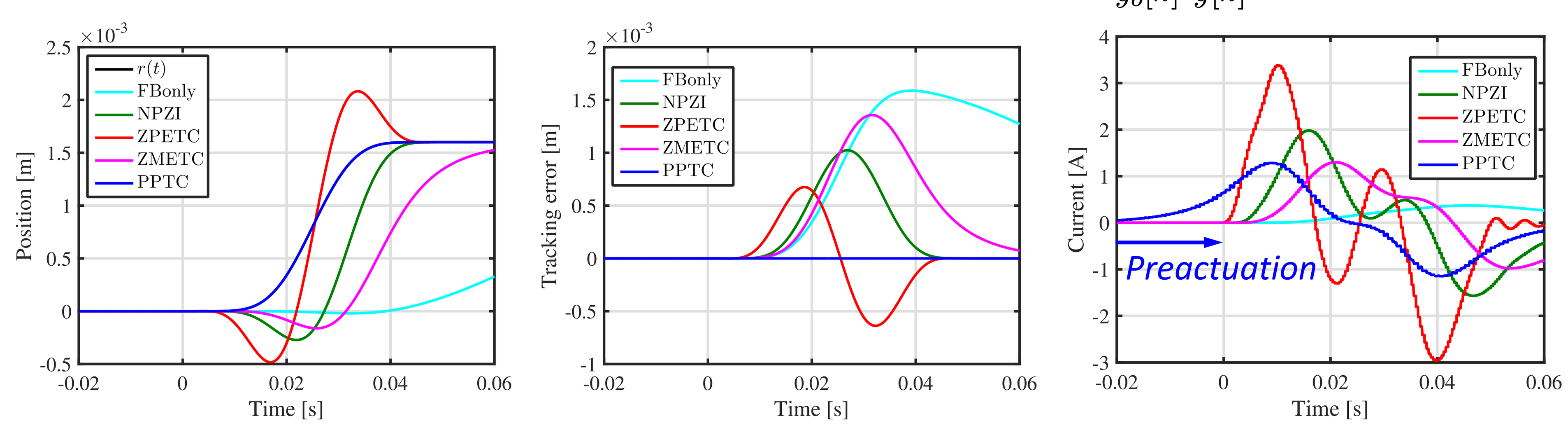
(c) Continuous pz-map
Unstable zero × 2



(d) Discrete pz-map
Unstable intrinsic zero × 2
Unstable discretization zero × 1

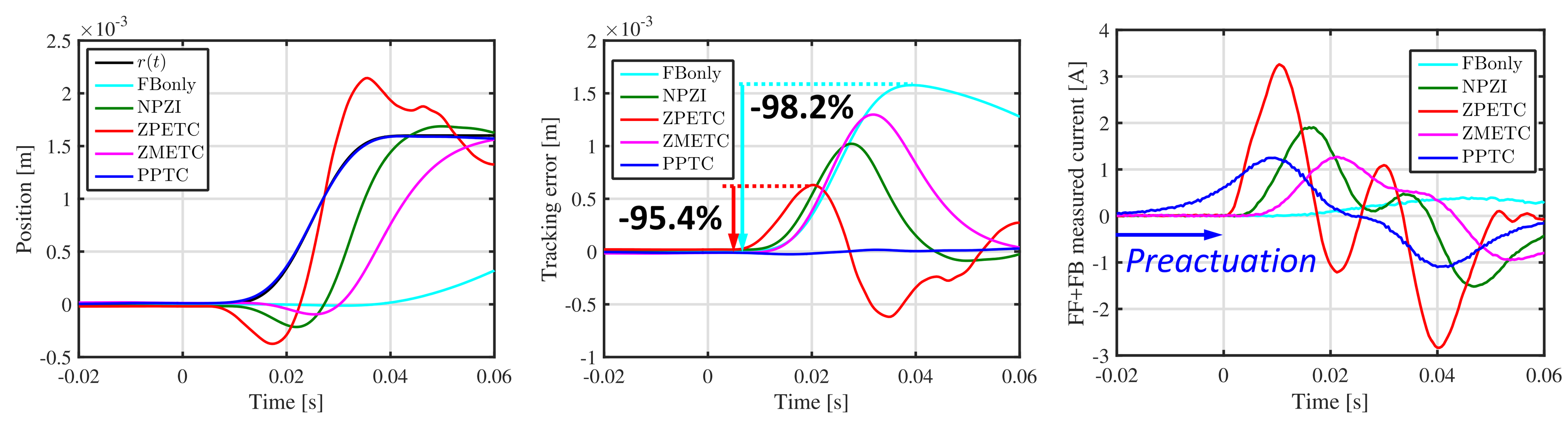
Modeling results using frequency domain identification





(a) Position. (b) Tracking error. (c) FF+FB current.

Simulation results without modeling errors or disturbances



(a) Position. (b) Tracking error. (c) FF+FB current.

Experimental results using 8th order feedforward

[W. Ohnishi, T. Beauduin, and H. Fujimoto, IFAC2017]